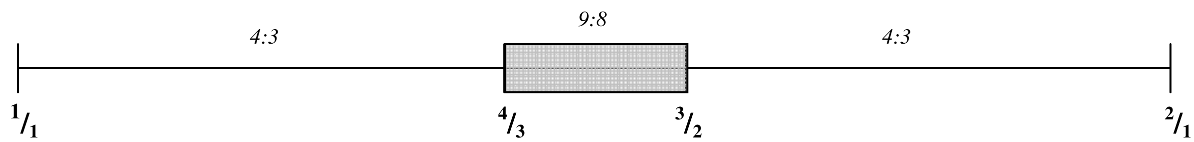
# Dissecting the Pitch Resources of *Objet Petit A*

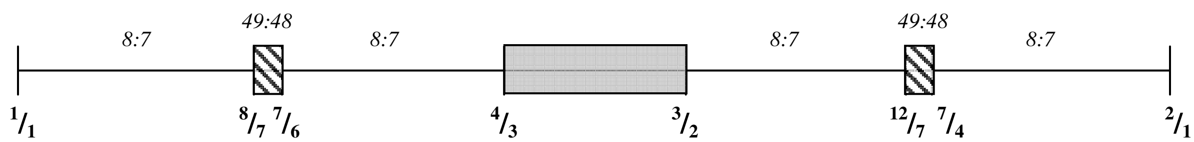
*Objet Petit A* (examples at: www.paulgreenhaw.com) is multi-movement work written for 2 electric organs and video projection. This paper examines the pitch resources used in the piece, followed by some further ruminations.

The octave (2:1) is divided by a series of concentric harmonic means (2AB/A+B):

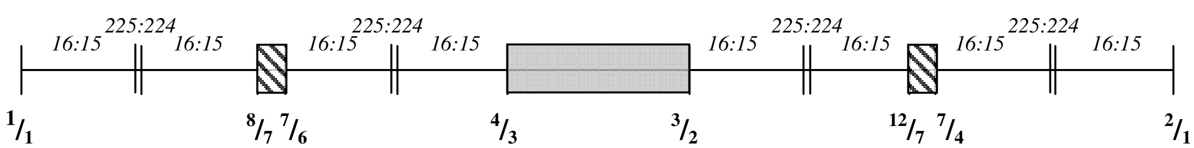
- The harmonic mean of 2:1 is 4:3.



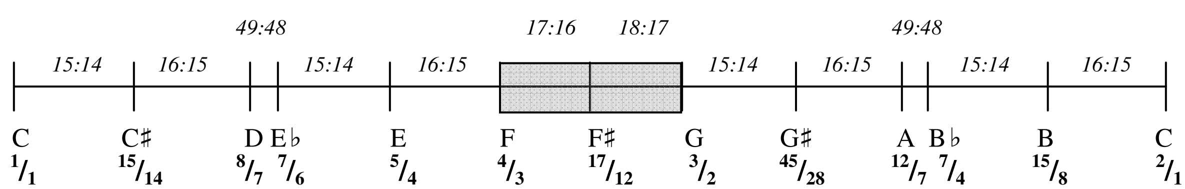

- The harmonic mean of 4:3 is 8:7.



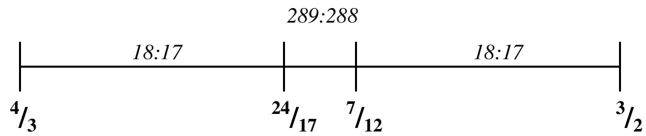

- The harmonic mean of 8:7 is 16:15.




225:224 (≈ 7.7 cents) is absorbed into a neighboring 16:15 – thus giving rise to pairs of 15:14 and 16:15 intervals.



As seen above, the central disjunctive tone, 9:8, has also been divided harmonically:




With 289:288 being absorbed into 18:17 – thus giving rise to a 17:16.

Notice every interval in the resultant scale is superparticular.

The octave has now been divided into 12 steps and can be easily mapped onto any standard keyboard.

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After living with this tuning for sometime, I found the pitch *D* ($^{8}/_{7}$) revealing itself as the gravitational center of the system. Further meditation on the scale revealed a possible (and logical) reason for this.

In order to discern the system more clearly, rename the intervals as whole numbers, as opposed to ratios. Accepting the integer "2" as a unity – and therefore factored out forthright – we find 21 (7 × 3) as the least common denominator:

| C | C♯ | D | E♭ | E | F | F♯ | G | G♯ | A | B♭ | B | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **$^{1}/_{1}$** | **$^{15}/_{7}$** | **$^{1}/_{7}$** | **$^{7}/_{3}$** | **$^{5}/_{1}$** | **$^{1}/_{3}$** | **$^{17}/_{3}$** | **$^{3}/_{1}$** | **$^{45}/_{7}$** | **$^{3}/_{7}$** | **$^{7}/_{1}$** | **$^{15}/_{1}$** | **$^{1}/_{1}$** |
| ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ |
| $^{21}/_{21}$ | $^{45}/_{21}$ | $^{3}/_{21}$ | $^{49}/_{21}$ | $^{105}/_{21}$ | $^{7}/_{21}$ | $^{119}/_{21}$ | $^{63}/_{21}$ | $^{135}/_{21}$ | $^{9}/_{21}$ | $^{147}/_{21}$ | $^{315}/_{21}$ | $^{21}/_{21}$ |

The scale can now be expressed using integers:

C C♯ D E♭ E F F♯ G G♯ A B♭ B C
21 – 45 – 3 – 49 – 105 – 7 – 119 – 63 – 135 – 9 – 147 – 315 – 21

Each pitch now has a unique relationship with the "1" or the "Absolute" (unmanifest in this case). Rearranging the sequence, from lesser to greater, hierarchically ranks the pitches:

D F A C C♯ E♭ G E F♯ G♯ B♭ B
<u>3 – 7 – 9 – 21 – 45 – 49 – 63 – 105 – 119 – 135 – 147 – 315</u>

This is an extraction of various partials along the harmonic series (in this case between "3" and "315"). We encounter the addition of new pitches as way traverse up thru each octave into higher and higher partials (keeping in mind that multiples of 2 are identities – that is: 3, 6, 12, 24 are all manifestations of our "D").

*First octave*:

| D | D |
|---|---|
| 3 – | 6 |

*Second Octave*:

| D | F | A | D |
|---|---|---|---|
| 6 – | 7 – | 9 – | 12 |

*Third Octave*:

| D | F | A | C | D |
|---|---|---|---|---|
| 12 – | 14 – | 18 – | 21 – | 24 |

*Fourth Octave*:

| D | F | A | C | C♯ | D |
|---|---|---|---|---|---|
| 24 – | 28 – | 36 – | 42 – | 45 – | 48 |

*Fifth Octave*:

| D | E♭ | F | G | A | C | C♯ | D |
|---|---|---|---|---|---|---|---|
| 48 – | 49 – | 56 – | 63 – | 72 – | 84 – | 90 – | 96 |

*Sixth Octave*:

| D | E♭ | E | F | F♯ | G | G♯ | A | B♭ | C | C♯ | D |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 96 – | 98 – | 105 – | 112 – | 119 – | 126 – | 135 – | 144 – | 147 – | 168 – | 180 – | 192 |

*Seventh Octave*:

| D | E♭ | E | F | F♯ | G | G♯ | A | B♭ | B | C | C♯ | D |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 192 – | 196 – | 210 – | 224 – | 238 – | 258 – | 270 – | 288 – | 294 – | 315 – | 336 – | 360 – | 384 |

We obtain all 12 pitch identities upon reaching the 192 – 384 octave.

The notion of a "manifest" and "unmanifest" $^{1}/_{1}$ deserves further attention. Compare and ponder a few common examples:

<u>Archytas's Diatonic</u>:

$^{1}/_{1}$ – $^{28}/_{27}$ – $^{32}/_{27}$ – $^{4}/_{3}$ // $^{3}/_{2}$ – $^{14}/_{9}$ – $^{16}/_{9}$ – $^{2}/_{1}$

↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓

$^{27}/_{27}$ – $^{28}/_{27}$ – $^{32}/_{27}$ – $^{36}/_{27}$ // $^{81}/_{27}$ – $^{42}/_{27}$ – $^{48}/_{27}$ – $^{54}/_{27}$

↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓

27 – (28) – (32) – (36) 81 – (42) – (48) – (54)

7 1 9 21 3 27

Least common denominator is 27 after factoring out all integers of 2.

The resultant hierarchy is as such: <u>1 – 3 – 7 – 9 – 21 – 27 – 81</u>
1 is manifest.

<u>Didymus's Diatonic</u>:

$^{1}/_{1}$ – $^{16}/_{15}$ – $^{32}/_{27}$ – $^{4}/_{3}$ // $^{3}/_{2}$ – $^{8}/_{5}$ – $^{16}/_{9}$ – $^{2}/_{1}$

↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓

$^{135}/_{135}$ – $^{144}/_{135}$ – $^{160}/_{135}$ – $^{180}/_{135}$ // $^{405}/_{135}$ – $^{216}/_{135}$ – $^{240}/_{135}$ – $^{270}/_{135}$

↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓

| 135 | – (144) | – (160) | – (180) | 405 | – (216) | – (240) | – (270) |
|---|---|---|---|---|---|---|---|
| | 9 | 5 | 45 | | 27 | 15 | 135 |

Least common denominator is 135 after factoring out all integers of 2.

The resultant hierarchy is as such: <u>5 – 9 – 15 – 27 – 45 – 135 – 405</u>
1 is not manifest in this case.

<u>Ptolemy's Intense Diatonic</u>:

$^{1}/_{1}$ – $^{16}/_{15}$ – $^{6}/_{5}$ – $^{4}/_{3}$ // $^{3}/_{2}$ – $^{8}/_{5}$ – $^{9}/_{5}$ – $^{2}/_{1}$

↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓

$^{15}/_{15}$ – $^{16}/_{15}$ – $^{18}/_{15}$ – $^{20}/_{15}$ // $^{45}/_{15}$ – $^{24}/_{15}$ – $^{27}/_{15}$ – $^{30}/_{15}$

↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓

| 15 | – (16) | – (18) | – (20) | 45 | – (24) | – 27 | – (30) |
|---|---|---|---|---|---|---|---|
| | 1 | 9 | 5 | | 3 | | 15 |

Least common denominator is 15 after factoring out all integers of 2.

The resultant hierarchy is as such: <u>1 – 3 – 5 – 9 – 15 – 27 – 45</u>
1 is unmanifest.

Ptolemy's Equable Diatonic:

Least common denominator is 165 after factoring out all integers of 2.

$^{1}/_{1}$ – $^{12}/_{11}$ – $^{6}/_{5}$ – $^{4}/_{3}$ // $^{3}/_{2}$ – $^{18}/_{11}$ – $^{9}/_{5}$ – $^{2}/_{1}$

↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓

$^{165}/_{165}$ – $^{180}/_{165}$ – $^{198}/_{165}$ – $^{220}/_{165}$ // $^{495}/_{165}$ – $^{270}/_{165}$ – $^{297}/_{165}$ – $^{270}/_{165}$

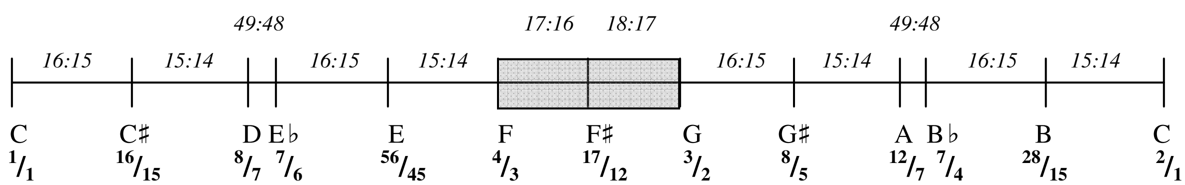
↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓

165 – (180)–(198) – (220)   495 – (270) – 297– (330)

45   99   55   135   165

The resultant hierarchy is as such: 45 – 55 – 99 – 135 – 165 – 297 – 495
1 is unmanifest.

Note the overall distance from 1 of Ptolemy's Equable Diatonic. This might be seen as a less harmonious pitch-set when compared to, say, Didymus's Diatonic (which sits lower in the harmonic series). Key decisions in creating *Objet Petit A*'s pitch-set were accomplished using this exact system of evaluation. Take, for instance, the four 8:7 intervals of the pitch-set; each is divided into a 15:14 and a 16:15. But what if the ordering was changed as such:




Here the least common denominator is 315, with a resultant integer series of:

315 – (336) – (360) – 735 – (392) – (420) – 1785 – 945 – 63 – (540) – 2205 – (588) – (630)

21   45   49   105   135   147   315

or hierarchically ordered:

21 – 45 – 49 – 63 – 105 – 135 – 147 – 315 – 735 – 945 – 1785 – 2205

Obliviously, not as "harmonious" as the previous design – in fact, no other permutation can sit as low in the harmonic series as the original. This is because the "moveable" pitches C♯, E, F♯, G♯, and B (the pitches who's location was determined when the 225:224 was absorbed) will create the most harmonious system when they add no new prime numbers to the denominators. Pitches C, D, E♭, F, G, A, and B♭ are, on the other hand, all fixed, and generate a least common denominator of 21 (i.e. primes 3 and 7). To keep the pitch-set from becoming less harmonious upon adding the moveable pitches, we need to keep from adding any new primes into the denominators – which, as seen in the original pitch-set, is completely possible.

Something rather beautiful is unraveling here – the notion that $^1/_1$, while not manifest in the actual pitch-set, is implied nevertheless. A "3" and a "5" inescapably point to a "1." Further inquiries into scales/tetrachords and their unmanifest, absolute "1" are necessary.

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